

Perturbative unitarity of Higgs derivative interactions

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Abstract

We study the perturbative unitarity bound given by dimension six derivative interactions consisting of Higgs doublets. These operators emerge from kinetic terms of composite Higgs models or integrating out heavy particles that interact with Higgs doublets. They lead to new phenomena beyond the Standard Model.

One of characteristic contributions by derivative interactions appear in vector boson scattering processes. Longitudinal modes of massive vector bosons can be regarded as Nambu Goldstone bosons eaten by each vector field with the equivalence theorem. Since their effects become larger and larger as the collision energy of vector bosons increases, vector boson scattering processes become important in a high energy region around the TeV scale. On the other hand, in such a high energy region, we have to take the unitarity of amplitudes into account.

We have obtained the unitarity condition in terms of the parameter included in the effective Lagrangian for one Higgs doublet models. Applying it to some of models, we have found that contributions of derivative interactions are not so large enough to clearly discriminate them from the Standard Model ones. We also study it in two Higgs doublet models. Because they are too complex to obtain the bound in the general effective Lagrangian, we have calculated it in explicit models. These analyses tell us highly model dependence of the perturbative unitarity bounds.

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1 Introduction

Almost all experimental results are consistent to what the Standard Model (SM) predicts. The recent observation of a new boson [1] have been also predicted by the model as the Higgs boson. Measured properties of the observed particle are still consistent to predicted ones, so that the SM is established by the observation. However, the model seems to be the effective theory describing physics below the electroweak (EW) scale since it includes several theoretical and experimental problems maybe explained at the TeV scale.

One of those mysteries is how to break the EW gauge symmetry. Since the symmetry is broken by hand in the SM, the model tells us what happens in experiments up to $O(100)$ GeV but does not tell us why and how the EW symmetry is broken. Therefore, we expect the SM should be extended to a model including other sectors responsible for physics of the electroweak symmetry breaking (EWSB) and other veiled phenomena. If the scale is much higher than the EW scale, the observed Higgs mass requires subtle mechanisms or fine tuning. Therefore we expect something new appears at several TeV region. Eventually, new structure is expected to appear at the TeV scale in order to obtain the EW scale without those artificial constructions.

Many of models describing physics beyond the SM have been proposed. Higgs sectors of these models are extended, and they break the EW symmetry with a certain mechanism. Effects of these new sectors could be firstly observed as deviations from the SM ones via higher dimensional operators at the scale lower than their original scales.

When we consider physics beyond the SM with extended Higgs sector, its low energy effective theory probably include dimension six derivative interactions as a part of higher dimensional operators. These operators have two origins: expansion of their kinetic terms if the Higgs doublet is realized as a part of pseudo Nambu Goldstone (NG) field; integrating out of heavy new scalar/vector bosons that interact with the Higgs field. The latter case appears even in models including elementary Higgs.

If the Higgs boson were removed from the SM, the Higgs sector would be described by $SU(2)_L \times SU(2)_R/SU(2)_V$ nonlinear sigma model. Derivative interactions of NG fields emerge from the kinetic term. These interactions contribute to longitudinal mode scatterings of massive gauge bosons with the equivalence theorem and cross sections of these processes become larger and larger as energy increases. They finally become so large as to violate the perturbative unitarity about 1 TeV [2]. Of course, the recent observation of the Higgs boson told us absence of the unitarity violation and reliability of the SM description even much above the TeV scale. We confront the similar problem in studying derivative interactions of Higgs doublets. In this

paper, we clarify scales where the given perturbative description is available with derivative interactions of the Higgs doublet in several models.

The rest of this paper is organized as follows. In Sec. 2, we study the unitarity bound given by derivative interactions in one Higgs doublet models (1HDMs). It is extended to the case of the two Higgs doublet models (2HDMs) in Sec. 3. In both of these sections, the unitarity violation scales are explicitly calculated with several models. Finally, our study is concluded in Sec. 4.

2 Unitarity of derivative interactions on one Higgs doublet models

The perturbative unitarity bounds given by the derivative interaction are discussed on 1HDMs. Firstly, we derive the formula of the unitarity bound and investigate its general properties. Then, results are applied to explicit models. The formulae of the perturbative unitarity are shown in App. A.

2.1 Formulae and general properties of the unitarity bound

The effective Lagrangian of derivative interactions in 1HDM is³

$$\mathcal{L} \supset \frac{c^H}{2f^2} \partial(H^\dagger H) \partial(H^\dagger H) + \frac{c^T}{2f^2} (H^\dagger \overleftrightarrow{\partial} H) (H^\dagger \overleftrightarrow{\partial} H), \quad (2.1)$$

where f is a scale related to new physics. The second operator should include covariant derivatives. However, gauge fields introduced by this kind of operator are not studied in this paper so that covariant derivatives are replaced with partial derivatives. Since the latter term violates the custodial symmetry, our analysis is based on the Lagrangian with $c^T = 0$.

Since we consider only four point scatterings given by Eq. (2.1), the vacuum expectation value of the Higgs boson play no role in the following calculation. Therefore, we use

$$H = \begin{pmatrix} C^+ \\ N \end{pmatrix}, \quad H^\dagger = (C^- \ N^\dagger), \quad (2.2)$$

where C^+/N is a charged/complex-neutral scalar field. The charged scalar and imaginary part of the neutral scalar are respectively eaten by W^\pm and Z bosons. Using the above notation, the following amplitudes are obtained⁴:

$$\begin{aligned} \mathcal{M}(C^+ C^- \rightarrow C^+ C^-) &= \mathcal{M}(NN^\dagger \rightarrow NN^\dagger) \\ &= \frac{\hat{s} + \hat{t}}{f^2} c^H, \end{aligned} \quad (2.3)$$

$$\mathcal{M}(C^+ C^- \rightarrow NN^\dagger) = \frac{\hat{s}}{f^2} c^H, \quad (2.4)$$

where \hat{s} and \hat{t} are the Mandelstam variables and particles are considered as massless, i.e., $\hat{s} + \hat{t} + \hat{u} = 0$.

Following Ref. [2], we construct matrices with partial wave amplitudes. The largest eigenvalue of these matrices give us the strongest bound to the perturbative unitarity. We have found zeroth modes produce the strongest bound in 1HDM, so that we focus on the case. With the formulae in App. A, the bound is given by the largest eigenvalue of the following matrix:

$$\begin{pmatrix} M_0(C^+ C^- \rightarrow C^+ C^-) & M_0(C^+ C^- \rightarrow NN^\dagger) \\ M_0(NN^\dagger \rightarrow C^+ C^-) & M_0(NN^\dagger \rightarrow NN^\dagger) \end{pmatrix} = \frac{\hat{s}}{16\pi f^2} \begin{pmatrix} c^H/2 & c^H \\ c^H & c^H/2 \end{pmatrix}, \quad (2.5)$$

namely, the bound is

$$\frac{\hat{s}}{f^2} \lesssim \frac{16\pi}{3c^H}. \quad (2.6)$$

³Using the field redefinition $H \rightarrow H + (a/f^2)(H^\dagger H)H$, where a is chosen as an appropriate value, any other dimension six derivative interaction of the Higgs doublet can be expressed with these kinds of operators given here [3].

⁴The EWSB produces corrections of $O(v^2/f^2)$. However we neglect them as neglecting $O(m_{W,Z}^2/E^2)$ corrections associated with the equivalence theorem and dimension eight operators because they are subleading effects in this study.

Process	Full	Central
$W_L^+ W_L^- \rightarrow hh$	1	1/2
$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$	2/3	13/48
$W_L^+ W_L^- \rightarrow Z_L Z_L$	1	1/2
$Z_L Z_L \rightarrow hh$	1	1/2
$Z_L Z_L \rightarrow W_L^+ W_L^-$	2	1
$W_L^+ Z_L \rightarrow W_L^+ Z_L$	2/3	13/48
$W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$	1	1/2

Table 1: Cross sections for vector boson scattering (VBS) processes in the unit of $\sigma(W_L^+ W_L^- \rightarrow hh)$. In the column of Full/Central, cross sections of VBS sub processes with/without the central region cut are shown.

Assuming that derivative interactions are purely given by kinetic term of the nonlinear sigma model, the conservative cut off scale is expressed in terms of the decay constant, i.e. $\Lambda \sim 4\pi f$ ⁵. Using the relation, the unitarity bound is related to the cut off scale as

$$\frac{\hat{s}}{\Lambda^2} \sim \frac{1}{3\pi c^H}. \quad (2.7)$$

Therefore, if the relation

$$c^H \lesssim \frac{1}{3\pi} \quad (2.8)$$

is satisfied, models reach the cut off scale before accessing the unitarity violation scale. Then, the effective Lagrangian, Eq. (2.1), is available up to the cut off scale. On the other hand, if the coefficient, c^H , is much larger than unity, the unitarity violation scale becomes comparable to the scale of new physics, f , so that the description of the effective Lagrangian is invalid even in the energy region around f . In the case c^H is $O(1)$, the unitarity violation scale exists between the scale of new physics and the cut off. Most of examples shown later are involved in this case. Around the unitarity bound, we have to include resonance effects, see, e.g., Ref [5]. It is therefore necessary to clarify valid energy scales for the description on each model.

We apply the result to cross sections of the Higgs boson and longitudinal modes of massive gauge bosons with the equivalence theorem. Since these scatterings are dominated by the coefficient, c^H , with the custodial symmetry, all of cross sections are proportional to each other. Here, we focus on only the process $W_L^+ W_L^- \rightarrow hh$, and relations with the others are shown in Tab. 1. Considering this sort of processes, the importance of the central region⁶ has been pointed out in Ref. [6], so that we also show ratios between the cross section of the Higgs pair production and those of the other processes with the central region cut. The cross section of $W_L^+ W_L^- \rightarrow hh$ is

$$\sigma(W_L^+ W_L^- \rightarrow hh) = \frac{\hat{s}}{32\pi} \left(\frac{c^H}{f^2} \right)^2 \lesssim \frac{8\pi}{9\hat{s}} \simeq \frac{1.1 \times 10^6}{\hat{s} [\text{TeV}]^2} [\text{fb}]. \quad (2.9)$$

For the process, Fig. 2.1 shows the region where the perturbative unitarity is violated .

Assuming that cross sections reach the above bound at $\sqrt{\hat{s}} = 3 \text{ TeV}$, the decay constant is

$$\frac{f}{\sqrt{c^H}} \sim \sqrt{\frac{3\hat{s}}{16\pi}} \sim 733 [\text{GeV}]. \quad (2.10)$$

If $c^H \sim 1$, the effect of the derivative interaction in the process becomes comparable with the SM one about $\sqrt{\hat{s}} = 2 \text{ TeV}$, where the cross section is $3 \times 10^4 \text{ fb}$ without the central region cut, see Ref. [6]. The value of f is typically related to new particle masses. For example, in the little Higgs scenario [7], the top partner mass is given by $O(f)$. From the viewpoint of the fine tuning, f is required to be below about 1 TeV.

⁵If UV completions are specified, the generalized dimensional analysis may produces lower cut off scales [4].

⁶This region is defined as $\cos \theta \in [-1/2, 1/2]$ in detectors, where θ is an angle from the beam axis.

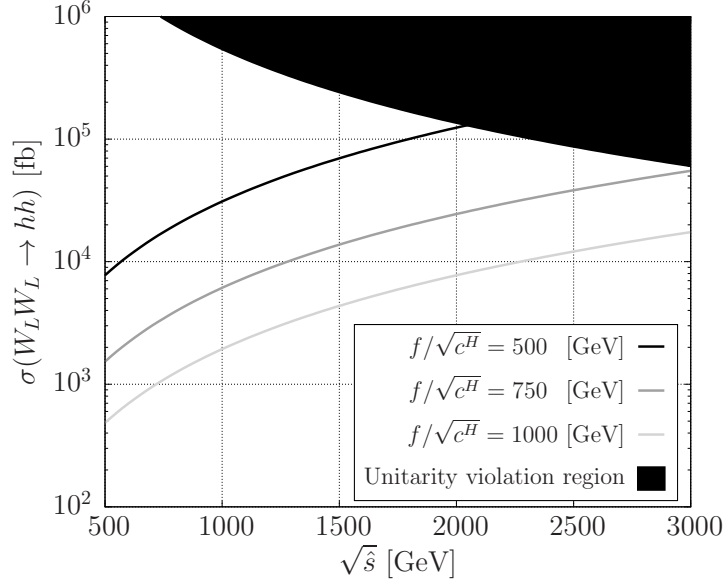


Figure 2.1: The upper bound of the cross section for $W_L^+ W_L^- \rightarrow hh$ with the perturbative unitarity condition. The horizontal axis is the collision energy of this VBS sub process. In the upper shaded region, the unitarity is broken down. The black, dark gray and light gray lines are the cross sections where $f/\sqrt{c^H} = 500, 750$ and 1000 GeV, respectively. For the other processes, the bounds can be obtained with the shift of the vertical axis by factors given in Tab. 1.

2.2 Examples with explicit models

In the rest of this section, we study the unitarity bounds on two models: the minimal composite Higgs model [8]; the littlest Higgs model with T-parity [9]. The latter model is studied in Refs. [10, 11]. In Ref. [11], several Little Higgs models are also investigated⁷. Since the normalization of decay constants can be changed, the combination f^2/c^H is meaningful. We here follow the normalization given in original papers. Decay constants have physical meanings through masses of additional massive vector bosons and fermions in each model.

The Higgs doublet is embedded so as to reflect the custodial symmetry. Therefore the operator violating the symmetry does not appear in both models.

2.2.1 The minimal composite Higgs model

This model is described by $SO(5)/SO(4)$ nonlinear sigma model including four NG fields. They are identified as the Higgs doublet.

The Lagrangian is

$$\mathcal{L} = \frac{f^2}{2} (\partial \Sigma)^\dagger (\partial \Sigma), \quad (2.11)$$

with

$$\Sigma = \begin{pmatrix} \sin[h/f] \vec{h}/h \\ \cos[h/f] \end{pmatrix}. \quad (2.12)$$

where \vec{h} is the real scalar multiplet of four NG bosons and h is its norm. Expanding these trigonometric functions, it is obtained that

$$c^H = 1. \quad (2.13)$$

⁷They obtained the unitarity bounds with all NG bosons. On the other hand, we focus on the Higgs doublets because other NG bosons are too heavy to treat as massless particles. Our results are conservative compared to theirs.

With the Eq. (2.6), the relation between the decay constant and the scale of the unitarity violation is

$$\frac{\hat{s}}{f^2} \sim \frac{16\pi}{3}. \quad (2.14)$$

Assuming that the perturbative unitarity is violated at 3 TeV, the decay constant is about 750 GeV. On the other hand, if the decay constant is chosen as 500 GeV, the perturbativity is preserved up to about 2 TeV, where the cross section of $W_L^+ W_L^- \rightarrow hh$ is 7×10^5 fb. In the case, the cross section of the Higgs boson pair production is one order of magnitude larger than that given by the SM. However, it is challenging to observe this process because the main decay mode is into four bottom state suffered from QCD background.

2.2.2 The littlest Higgs model with T-parity

Derivative interactions on the littlest Higgs model with T-parity is shown below. The scalar fields are described by $SU(5)/SO(5)$ nonlinear sigma model which include 14 NG bosons.

The kinetic term of this model is

$$\mathcal{L} = \frac{f^2}{8} \text{tr} \left[\left(\partial e^{-2i\Pi/f} \right) \left(\partial e^{2i\Pi/f} \right) \right], \quad (2.15)$$

where Π is NG field. The Higgs doublet is assigned in the NG field as

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} H & \\ H^\dagger & H^T \\ & H^* \end{pmatrix}. \quad (2.16)$$

We omitted the other NG bosons since they don't contribute to the current analysis. Extracting the derivative interaction from the kinetic term, we obtain

$$c^H = \frac{1}{2}. \quad (2.17)$$

This result is consistent with previous works [10, 11].

If we suppose that $f = 750$ GeV, the perturbative unitarity is preserved up to about 4 TeV. Hence this model description is valid in higher energy scale while signals of the derivative interaction are smaller than the previous model. For $f = 750$ GeV, the cross section of $W_L^+ W_L^- \rightarrow hh$ in this model is almost corresponding to the line where $f/\sqrt{c^H}$ is 1000 GeV.

3 Unitarity of derivative interactions on two Higgs doublet models

In this section we extend the previous discussion to dimension six derivative interactions including two Higgs doublets.

The modification is straightforward, and the prescription is also simple. However, formulae become too complex because of many degrees of freedom (DOF). Then we cannot obtain the formula of the strongest bound like Eq. (2.6) with the largest eigenvalue of a matrix that consists of partial wave amplitudes. Since the matrix can be diagonalized in individual models, three models are shown as examples.

In this section, we consider processes whose initial states are electromagnetic neutral. Matrices producing the unitarity bounds for singly or doubly charged initial states are also shown in App. C.

3.1 Formulae and general properties of the unitarity bound

The analyses in this section are based on the following effective Lagrangian:

$$\begin{aligned}\mathcal{L} \supset & \frac{c_{1111}^H}{f^2} O_{1111}^H + \frac{c_{1112}^H}{f^2} (O_{1112}^H + O_{1121}^H) \\ & + \frac{c_{1122}^H}{f^2} O_{1122}^H + \frac{c_{1221}^H}{f^2} O_{1221}^H + \frac{c_{1212}^H}{f^2} (O_{1212}^H + O_{2121}^H) \\ & + \frac{c_{2221}^H}{f^2} (O_{2221}^H + O_{2212}^H) + \frac{c_{2222}^H}{f^2} O_{2222}^H \\ & + \frac{c_{1122}^T}{f^2} O_{1122}^T + \frac{c_{1221}^T}{f^2} O_{1221}^T + \frac{c_{1212}^T}{f^2} (O_{1212}^T + O_{2121}^T),\end{aligned}\tag{3.1}$$

where

$$O_{ijkl}^H = \frac{1}{1 + \delta_{ik}\delta_{jl}} \partial(H_i^\dagger H_j) \partial(H_k^\dagger H_l),\tag{3.2}$$

$$O_{ijkl}^T = \frac{1}{1 + \delta_{ik}\delta_{jl}} (H_i^\dagger \overleftrightarrow{\partial} H_j) (H_k^\dagger \overleftrightarrow{\partial} H_l),\tag{3.3}$$

and

$$H_i = \begin{pmatrix} C_i^+ \\ N_i \end{pmatrix}, \quad H_i^\dagger = (C_i^- \ N_i^\dagger).\tag{3.4}$$

To impose the custodial symmetry, the above coefficients are real and follow relations derived in App. B:

$$3c_{1122}^T + c_{1221}^H - c_{1212}^H = 0,\tag{3.5}$$

$$c_{1122}^T + c_{1221}^T + c_{1212}^T = 0.\tag{3.6}$$

The 2HDMs require mixing angles to get mass eigenstates of scalar fields. In this paper, we use the equivalence theorem and focus on only derivative interactions, that is, masses of scalar fields are neglected. In this case, the perturbative unitarity bound is independent of mixing angles. This is also true for models including N Higgs doublets.

Finally, the unitarity bound is

$$\frac{\hat{s}}{f^2} \lesssim \frac{8\pi}{|C_{\max}|},\tag{3.7}$$

where C_{\max} is the largest eigenvalue of the matrices given in App. C ⁸.

As we will see later, the largest eigenvalue $|C_{\max}|$ can be as large as about ten. In this case, the unitarity bound becomes quite strong and leads to an interesting remark. Suppose that, for instance, the pair production of a heavy particle whose mass is $O(f)$ in VBS processes, the energy scale where the pair is produced could be as large as the unitarity violation scale. This means we couldn't discuss the kind of process with these low energy descriptions.

3.2 Examples with explicit models

We study consequences of the above result with several models including two Higgs doublets. Following three models are studied: the bestest little Higgs model [12]; the UV friendly little Higgs model [13]; an inert doublet model. The first and second ones are composite Higgs models and the last one is a toy model including elementary Higgs doublets.

⁸Eq. (3.7) and the viewpoint explained below are also stated in Ref. [11].

3.2.1 The bestest little Higgs model

The bestest little Higgs model is a little Higgs model which includes two Higgs doublets. Scalar fields are implemented as $SO(6) \times SO(6)/SO(6)$ nonlinear sigma model including 15 NG bosons. The normalization of the kinetic term is the same as Eq. (2.15), and the NG field is

$$\Pi = \frac{i}{\sqrt{2}} \begin{pmatrix} h_1 & h_2 \\ -h_1^T & -h_2^T \end{pmatrix}, \quad (3.8)$$

where $h_{1,2}$ are real scalar multiplets considered two Higgs doublets and the other NG bosons are eliminated. In this model, Higgs doublets interact with heavy gauge bosons and a singlet scalar. The masses of heavy gauge bosons depend on the other decay constant that is larger than f in order to avoid constraints from the electroweak precision measurement (EWPM). Thus effects coming from the heavy gauge bosons are tiny, then we neglect them. The interaction with a singlet is required to obtain a collective quartic coupling. For simplicity, we introduce the following terms to see the effect:

$$\mathcal{L}_\sigma \supset -\frac{m_\sigma^2}{2}\sigma^2 + \lambda f \sigma (H_1^\dagger H_2 + \text{H.c.}), \quad (3.9)$$

where σ is a neutral singlet scalar⁹. Including this contribution, coefficients of derivative interactions are

$$c_{1111}^H = \frac{1}{2}, \quad c_{1112}^H = 0, \quad (3.10)$$

$$c_{1122}^H = 0, \quad c_{1221}^H = \frac{1}{4} + c^\sigma, \quad c_{1212}^H = \frac{1}{4} + c^\sigma, \quad (3.11)$$

$$c_{2221}^H = 0, \quad c_{2222}^H = \frac{1}{2}, \quad (3.12)$$

$$c_{1122}^T = 0, \quad c_{1221}^T = \frac{1}{4}, \quad c_{1212}^T = -\frac{1}{4}, \quad (3.13)$$

where

$$c^\sigma = \frac{\lambda^2 f^4}{m_\sigma^4}. \quad (3.14)$$

The unitarity bound depends on the value of c^σ because the largest eigenvalue is a function of it. For $0 \leq c^\sigma < 1/8$, the bound is

$$\frac{\hat{s}}{f^2} \lesssim \frac{16\pi}{2 - c^\sigma}. \quad (3.15)$$

At $c^\sigma = 0$, it is bounded as

$$\frac{\hat{s}}{f^2} \lesssim 8\pi, \quad (3.16)$$

and it becomes weak as c^σ increases. At $c^\sigma = 1/8$, the bound is the weakest:

$$\frac{\hat{s}}{f^2} \lesssim 8 \frac{16\pi}{15}. \quad (3.17)$$

In the region, $1/8 < c^\sigma$, the bound is

$$\frac{\hat{s}}{f^2} \lesssim \frac{16\pi}{1 + 7c^\sigma}, \quad (3.18)$$

where the right hand side decrease as c^σ increases and the bound becomes the same as the case of $c^\sigma = 0$ at $c^\sigma = 1/7$.

⁹In the original paper [12], $m_\sigma = \sqrt{\lambda_{65} + \lambda_{56}}f$ and $\lambda = \frac{\lambda_{65} - \lambda_{56}}{\sqrt{2}}$.

The unitarity bounds of $W_L^+ W_L^- \rightarrow hh$ and $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$ are displayed below. We define mass eigenstates, h and W_L^\pm , as follows:

$$\frac{h}{\sqrt{2}} = N_1^R \cos \alpha + N_2^R \sin \alpha, \quad (3.19)$$

$$W_L^\pm = C_1^\pm \cos \beta + C_2^\pm \sin \beta, \quad (3.20)$$

where N_i^R is the real part of N_i and α and β are mixing angles. Unitarity bounds for these processes are

$$\sigma(W_L^+ W_L^- \rightarrow hh) = \frac{\hat{s}}{32\pi f^4} B_h(\alpha, \beta)^2 \lesssim \frac{2\pi B_h(\alpha, \beta)^2}{\hat{s} C_{\max}^2}, \quad (3.21)$$

$$\sigma(W_L^+ W_L^+ \rightarrow W_L^+ W_L^+) = \frac{\hat{s}}{32\pi f^4} B_w(\beta)^2 \lesssim \frac{2\pi B_w(\beta)^2}{\hat{s} C_{\max}^2}, \quad (3.22)$$

where

$$B_h(\alpha, \beta) = \frac{1}{4} (1 + (1 + 2c^\sigma(1 - c_{4\beta}))c_{2(\alpha-\beta)} + 2c^\sigma s_{4\beta}s_{2(\alpha-\beta)}), \quad (3.23)$$

$$B_w(\beta) = \frac{1}{2} (1 + c^\sigma(1 - c_{4\beta})). \quad (3.24)$$

Parameters c_x and s_x are $\cos x$ and $\sin x$, and $C_{\max} = (2 - c^\sigma)/2$ for $0 \leq c^\sigma < 1/8$ and $C_{\max} = (1 + 7c^\sigma)/2$ for $1/8 \leq c^\sigma$. If $\alpha = \beta$ is satisfied, it is called the decoupling limit, and we get the relation: $B_h(\beta, \beta) = B_w(\beta)$.

The perturbative unitarity bounds of $W_L^+ W_L^- \rightarrow hh$ are shown in Fig. 3.1. In order to see the effects of new parameters, we fix the decay constant as 750 GeV. The shaded regions in these figures are changed by the mixing angles because the cross section depends on the angles. However, the unitarity bound itself depends on only the coefficient, c^σ . Hence we can see that the energy scales where the cross sections intersect the unitarity violation regions are independent on the angles, e.g., $\sqrt{\hat{s}} \sim 1.9$ TeV for $c^\sigma = 1$. For $\beta = 0$ and $\alpha - \beta = \pi/6$, the cross sections don't depend on the value of c^σ ; thus, we have only one line but still intersecting points are the same.

3.2.2 The UV friendly little Higgs model

The UV friendly little Higgs model also includes two Higgs doublets in a part of 14 NG bosons given by $SU(6)/Sp(6)$ nonlinear sigma model. The normalization of the kinetic term is also the same as Eq. (2.15). Since we study only Higgs doublets, NG field Π can be considered as follows¹⁰:

$$\Pi = \frac{1}{2} \begin{pmatrix} -\varepsilon(H_1 - H_2) & H_1 + H_2 & -H_1^T - H_2^T \\ \varepsilon(H_1^\dagger - H_2^\dagger) & & \varepsilon(H_1^T - H_2^T) \\ H_1^\dagger + H_2^\dagger & -H_1^* - H_2^* & -\varepsilon(H_1^* - H_2^*) \end{pmatrix}, \quad (3.25)$$

where ε is the totally antisymmetric tensor, $\varepsilon^{12} = 1$. Since contributions given by heavy new particles can be ignored in this model, only derivative interactions generated by the kinetic term are taken into account. Following coefficients of derivative interactions are produced by the kinetic term:

$$c_{1111}^H = 4, \quad c_{1112}^H = 0, \quad (3.26)$$

$$c_{1122}^H = 1, \quad c_{1221}^H = 0, \quad c_{1212}^H = -3, \quad (3.27)$$

$$c_{2221}^H = 0, \quad c_{2222}^H = 4, \quad (3.28)$$

$$c_{1122}^T = 1, \quad c_{1221}^T = 0, \quad c_{1212}^T = 1. \quad (3.29)$$

¹⁰ Assignment and normalization of NG bosons given by the original paper are different from ordinary prescription of the nonlinear sigma model. Since we study only the part of two Higgs doublets, normalization of these field are changed in order to study with the canonical normalization.

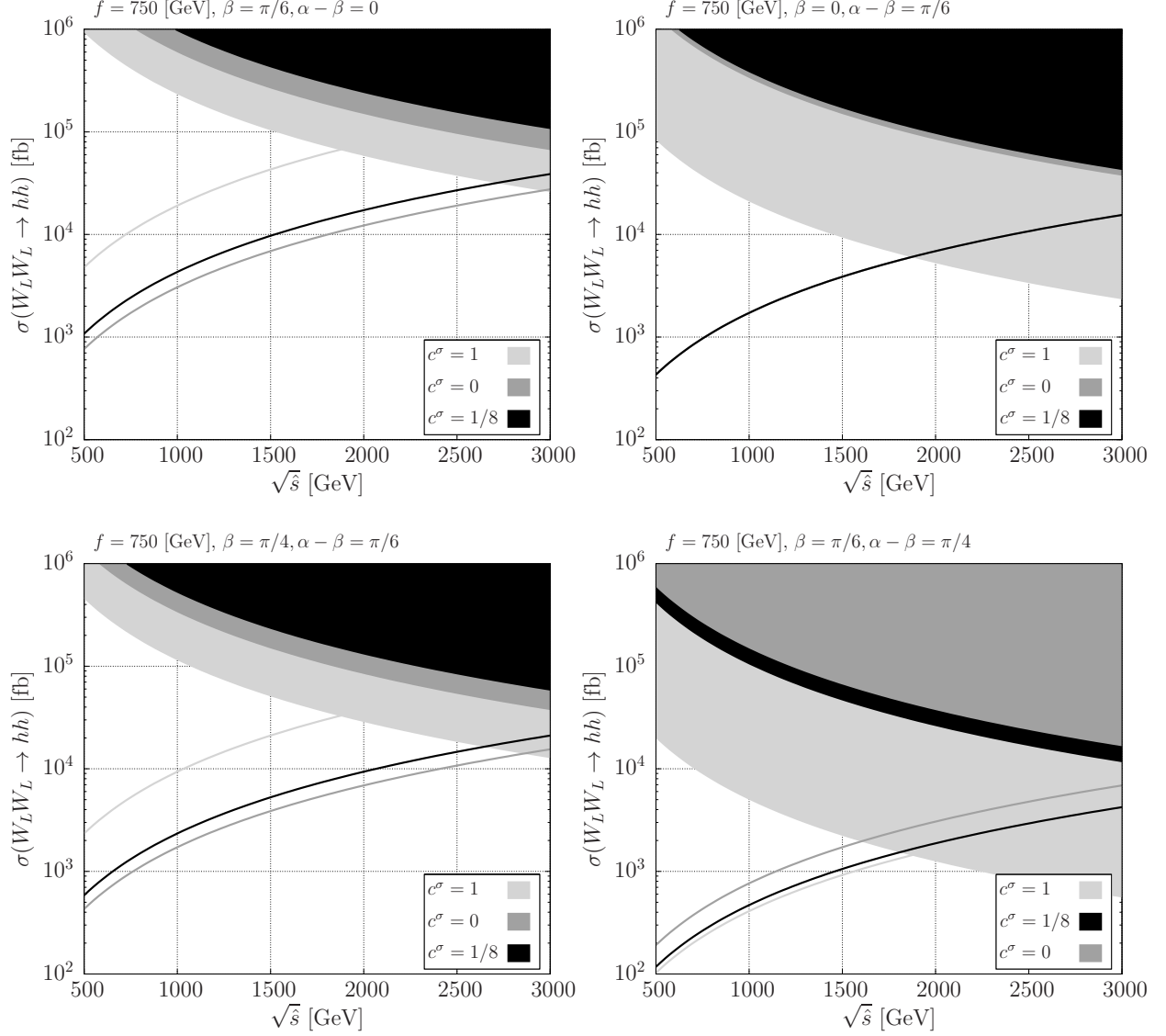


Figure 3.1: The perturbative unitarity bounds of $W_L^+ W_L^- \rightarrow hh$ for various c^σ and mixing angles. The decay constant is fixed as 750 GeV. The Mixing angles are set as $(\beta, \alpha - \beta) = (\pi/6, 0)$ (upper left), $(0, \pi/6)$ (upper right), $(\pi/4, \pi/6)$ (lower left) and $(\pi/6, \pi/4)$ (lower right). The light gray, dark gray and black lines are cross sections for $c^\sigma = 1, 0$ and $1/8$, respectively. Unitarity violation regions depend on the value of c^σ , and their brightness correspond to each line.

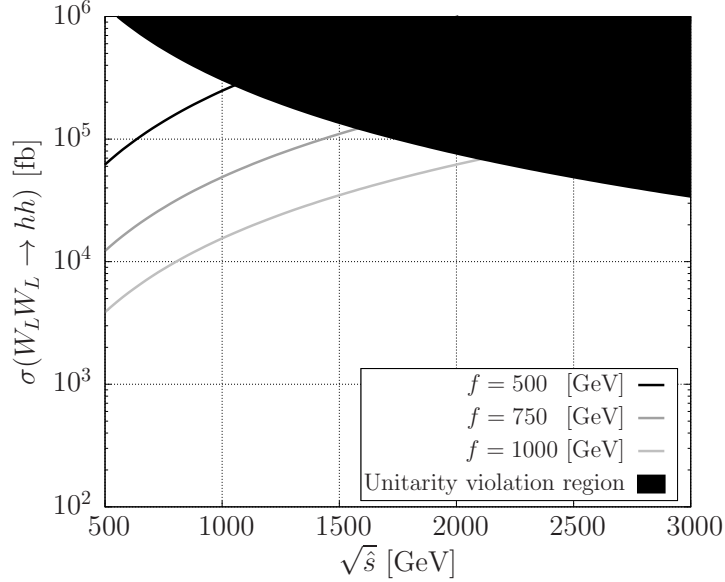


Figure 3.2: The perturbative unitarity bounds of $W_L^+ W_L^- \rightarrow hh$ in the UV friendly little Higgs model. The horizontal axis is the collision energy of this VBS sub process. In the upper shaded region, the unitarity is broken down. The black, dark gray and light gray lines are the cross sections corresponding to $f = 500, 750$ and 1000 GeV, respectively.

These coefficients apparently violate the custodial invariant conditions, Eqs. (3.5) and (3.6). However, tree level contributions to ρ parameter cannot occur because of Z_2 symmetry. With these coefficients, the strongest bound is

$$\frac{\hat{s}}{f^2} \lesssim \pi. \quad (3.30)$$

Assuming that the perturbative unitarity is violated at 3 TeV, the decay constant, f , is determined as 1.7 TeV. This value looks large in the viewpoint of the fine tuning as we have already seen. On the other hand, if the decay constant is about 1 TeV, the unitarity is broken below about 1.7 TeV.

The unitarity bounds of $W_L^+ W_L^- \rightarrow hh$ and $W_L^+ W_L^- \rightarrow W_L^+ W_L^+$ are

$$\sigma(W_L^+ W_L^- \rightarrow hh) = \frac{\hat{s}}{2\pi f^4} \lesssim \frac{\pi}{2\hat{s}}, \quad (3.31)$$

$$\sigma(W_L^+ W_L^- \rightarrow W_L^+ W_L^+) = \frac{\hat{s}}{2\pi f^4} \lesssim \frac{\pi}{2\hat{s}}. \quad (3.32)$$

Note that the cross sections have no mixing angle dependences because of Z_2 structure; only one Higgs doublet gets the vacuum expectation value. These bounds to cross sections are shown in Fig. 3.2. It corresponds to the case $c^\sigma = 15/7$ for the bestest little Higgs model. The unitarity bound of this model is severe because the largest eigenvalue is quite larger than the previous models.

3.2.3 Inert doublet models with odd scalars

We investigate the following Lagrangian consisting of elementary scalar and vector fields:

$$\begin{aligned}
\mathcal{L} \supset & -\frac{m_{s0}^2}{2}\phi_0^2 + \lambda_0 f \phi_0 \left(H_1^\dagger H_2 + \text{H.c.} \right) \\
& -\frac{m_{sL}^2}{2}\phi_L^a \phi_L^a + \lambda_L f \phi_L^a \left(H_1^\dagger \sigma^a H_2 + \text{H.c.} \right) \\
& -m_{sL}^2 \phi_T^{a\dagger} \phi_T^a + \sqrt{2}\lambda_L f \left(\phi_T^{a\dagger} (H_1^T \sigma^2 \sigma^a H_2) + \text{H.c.} \right) \\
& +\frac{m_{v0}^2}{2}V_0 \cdot V_0 + g_0 V_0 \cdot \left(iH_1^\dagger \overleftrightarrow{\partial} H_2 + \text{H.c.} \right) \\
& +m_{v0}^2 V_S^\dagger \cdot V_S + \sqrt{2}g_0 \left(iV_S^\dagger \cdot H_1^T \sigma^2 \overleftrightarrow{\partial} H_2 + \text{H.c.} \right) \\
& +\frac{m_{vL}^2}{2}V_L^a \cdot V_L^a + g_L V_L^a \cdot \left(iH_1^\dagger \sigma^a \overleftrightarrow{\partial} H_2 + \text{H.c.} \right). \tag{3.33}
\end{aligned}$$

Scalar fields, ϕ_0 , ϕ_L^a and ϕ_T^a , are respectively $\mathbf{1}_0$, $\mathbf{3}_0$ and $\mathbf{3}_1$ representations of $SU(2)_L \times U(1)_Y$, and vector fields, V_0 , V_S and V_L^a are respectively $\mathbf{1}_0$, $\mathbf{1}_1$ and $\mathbf{3}_0$ representations of the gauge symmetry. We suppose that these new particles and H_2 are odd under an additional Z_2 symmetry, and H_1 and the other SM particles are even under the discrete symmetry. These choices of couplings and masses for ϕ_L^a and ϕ_T^a , and V_0 and V_S are required to respect $SO(4)$ symmetry¹¹. This set up suppresses contributions to oblique corrections.

Integrating out heavy particles, following coefficients of derivative interactions are obtained:

$$c_{1111}^H = 0, \quad c_{1112}^H = 0, \tag{3.34}$$

$$c_{1122}^H = -2s_L + 3v_0 + 3v_L, \quad c_{1221}^H = s_0 - 2s_L + 3v_0, \quad c_{1212}^H = s_0 - 2s_L + 3v_L, \tag{3.35}$$

$$c_{2221}^H = 0, \quad c_{2222}^H = 0, \tag{3.36}$$

$$c_{1122}^T = -v_0 + v_L, \quad c_{1221}^T = -s_L - v_L, \quad c_{1212}^T = s_L + v_0, \tag{3.37}$$

where

$$s_0 = \left(\frac{\lambda_0 f^2}{m_{s0}^2} \right)^2, \quad s_L = \left(\frac{\lambda_L f^2}{m_{sL}^2} \right)^2, \tag{3.38}$$

$$v_0 = \left(\frac{g_0 f}{m_{v0}} \right)^2, \quad v_L = \left(\frac{g_L f}{m_{vL}} \right)^2. \tag{3.39}$$

Even if additional particle exist, their contributions are included into these four coefficients. We cannot discriminate these multiple contributions from a large contribution given by a particle. Using these coefficients, eigenvalues of the matrix (C.1) are fortunately obtained as the following simple forms:

$$c_1^I = -\frac{s_0 - 7s_L + 3v_0 + 3v_L}{2}, \tag{3.40}$$

$$c_2^I = \frac{7s_0 - 9s_L + 9v_0 + 9v_L}{2}, \tag{3.41}$$

$$c_3^I = \frac{s_0 - 3s_L + 3v_0 - 9v_L}{2}, \tag{3.42}$$

$$c_4^I = \frac{s_0 - 3s_L - 9v_0 + 3v_L}{2}, \tag{3.43}$$

$$c_{5\pm}^I = \pm \frac{s_0 + s_L + 3v_0 + 3v_L}{2}, \tag{3.44}$$

$$c_{6\pm}^I = \pm \frac{s_0 + 9s_L - 9v_0 - 9v_L}{2}. \tag{3.45}$$

¹¹This structure should be broken by renormalization group running of them even if the UV completion possesses the structure. We assume that this $SO(4)$ symmetry is still good symmetry so as to suppress large corrections to the ρ parameter in the scale where the Lagrangian (3.33) is available.

The strongest unitarity condition is also given by Eq. (3.7) with the largest eigenvalue in the above.

Cross sections of $W_L^+ W_L^- \rightarrow hh$ and $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ vanish because only a Higgs doublet has nonzero vacuum expectation value, and these interactions must include both of Z_2 even and odd particles.

In this model we have four coefficients Eqs. (3.38) and (3.39) to parametrize the dimension six differential operators. If we suppose that $s_0 = s_L = v_0 = v_L = 1$, the eigenvalue c_2^I becomes the largest: $c_2^I = 8$. This value gives us the perturbative unitarity condition as Eq. (3.30).

The unitarity bound, Eq. (3.7), can be interpreted as the perturbativity condition of couplings. For example, if $s_0 = s_L = v_0 = 0$ and $v_L \neq 0$, we get $|C_{\max}| = 9v_L/2$. Then, the unitarity bound can be expressed as

$$\sqrt{\hat{s}} \sim \frac{4\sqrt{\pi}}{3} \frac{m_{vL}}{g_L}. \quad (3.46)$$

In order to preserve unitarity, the unitarity violation scale should be larger than the mass, m_{vL} . As a result, we get the following condition:

$$g_L \lesssim \frac{4\sqrt{\pi}}{3}. \quad (3.47)$$

If the model includes only one $\mathbf{3}_0$ vector, this requirement is stronger than the naive perturbativity condition, $g_L < 4\pi$. On the other hand, if it includes several $\mathbf{3}_0$ vectors, the unitarity bound also limits the number of them.

4 Conclusion

We have studied the perturbative unitarity for dimension six derivative interactions of the Higgs doublets. They are generated by kinetic terms in composite Higgs models or integrating out of heavy particles which interact with the Higgs doublets. Because of the latter case, derivative interactions appear even in models consisting of elementary Higgs doublets.

We have firstly studied it in models including only one Higgs doublet. The strongest bounds for the unitarity are expressed by the largest eigenvalue of the matrix given by partial wave amplitudes of VBS processes. We focus on high energy region such that derivative interactions could dominate longitudinal mode contributions. Assuming that given derivative interactions respect the EWPM, only a combination of parameter, c^H/f^2 , appear in the analysis. Therefore, the unitarity condition is expressed by the parameter as Eq. (2.6). We have applied it to the cross section of $W_L W_L \rightarrow hh$.

We have calculated the bounds on explicit models: the minimal composite Higgs model; the littlest Higgs model. Structures of global symmetry are significantly different each other; $SO(5)/SO(4)$ and $SU(5)/SO(5)$. However, given bounds are similar; $c^H = 1$ and $1/2$. Decay constants f are related to masses of the top like fermions in composite Higgs models. Therefore it is supposed that $f/\sqrt{c^H}$ is larger than about 500 GeV, see e.g. Ref. [14], where the perturbative unitarity is violated above the region $\sqrt{\hat{s}} \gtrsim 2$ TeV. Even in this case, it is difficult to obtain large cross sections enough to distinguish new physics contributions from the SM ones.

Secondly, similar analyses have been performed in 2HDMs. A simple formula of the unitarity bound could not be obtained with parameters included in the effective Lagrangian (3.1) since the matrix of partial wave amplitudes are too complex to be diagonalized. Hence we have investigated the unitarity bound with explicit models: the bestest little Higgs model; the UV friendly little Higgs model; the inert doublet model. The first and the second ones are literally a kind of little Higgs model and the third one is a toy model including elementary Higgs doublets.

In the first one, derivative interactions are generated by not only the kinetic term but also integrating out of a heavy scalar field. Constraints of the former one to the unitarity is similar to those given by 1HDMs discussed in Sec. 2. Including the latter one, the largest eigenvalue depends on the scalar contribution. The unitarity bound can be stronger than the case not including it.

In the second model, the unitarity condition is much severe comparing to the other models mentioned in this paper. This is because coefficients of derivative interactions are large in this model. These large coefficients produce large cross sections, so that the unitarity bounds of cross sections are also large. However, energy scales where the unitarity is broken down depend on the largest eigenvalue. Therefore the unitarity is

violated at low scale compared with that of other models. In this kind of model, large cross sections, enough to overcome the SM ones, can be achieved even with the large decay constant if people discard the naturalness criterion. On the other hand, assuming that masses of additional particles are near the decay constant, they are also near the scale of the unitarity violation. Therefore, study of vector resonance is probably required when people investigate, for instance, pair productions of these additional particles with vector boson collisions.

In the last model which includes elementary Higgs doublet, VBSs of the SM particles are suppressed by the additional Z_2 symmetry. In this kind of model, masses of heavy particles should be smaller than the unitarity violation scale. This condition means that couplings between Higgs doublets and heavy particles are much smaller than the strong coupling, 4π , or the number of these particles are limited.

As a conclusion, we have clarified the importance to study the unitarity bound when Higgs derivative interactions are investigated since the bound can be significantly lower than the naive cut off scale.

Acknowledgement

We are grateful to Y. Okada for watching over our studies. The author Y.Y. would like to thank K. Hamaguchi, T. Moroi and M. Tanabashi for useful comments. The author Y.K. would like to thank S. Kanemura for useful discussions. The work of Y.Y. is supported in part by the Grant-in-Aid for Science Research, Japan Society for the Promotion of Science (JSPS), No. 22244021.

A Perturbative unitarity

Amplitudes of elastic scattering satisfy the following relation for each partial wave:

$$M_n^I = \lambda(a, b) |M_n^R + iM_n^I|^2, \quad (\text{A.1})$$

$$\lambda(a, b) = \sqrt{(1 - (a + b)^2)(1 - (a - b)^2)}, \quad (\text{A.2})$$

where M_n^R/M_n^I is the real/imaginary part of partial wave amplitudes, a and b are mass fractions between the mass of each particle and the center of mass energy, $m_{a,b}/\sqrt{s}$, and partial waves are defined as below with the Legendre polynomials, $P_m(x)$:

$$\mathcal{M}(\cos \theta) = 16\pi \sum_{n=0}^{\infty} (2n+1) M_n P_n(\cos \theta), \quad (\text{A.3})$$

$$\int_{-1}^1 dx P_m(x) P_n(x) = \frac{2}{2n+1} \delta_{mn}. \quad (\text{A.4})$$

Eq. (A.1) is the equation of circle whose radius is $1/2\lambda$ and the center is $(0, 1/2\lambda)$. With the high energy limit where masses of produced particles can be neglected, the radius of the circle becomes the maximum. Therefore, the actual amplitudes are included by the maximal circle. Finally, partial wave amplitudes at least satisfy $M_n^R \in [-1/2, 1/2]$ and $M_n^I \in [0, 1]$. If processes include the same particles in the final state, the bound becomes weaker as $M_n^R \in [-1, 1]$ and $M_n^I \in [0, 2]$.

Considering the unitarity bound for derivative interactions, with the massless limit, amplitudes of derivative interactions can be written as

$$\mathcal{M} = \frac{C_s \hat{s} + C_t \hat{t}}{f^2} \quad (\text{A.5})$$

$$= \frac{\hat{s}}{f^2} \left(\left(C_s - \frac{C_t}{2} \right) P_0 + \frac{C_t}{2} P_1 \right), \quad (\text{A.6})$$

where s and t are the Mandelstam variables. The zeroth and the first modes of partial wave amplitudes appear:

$$M_0 = \frac{\hat{s}}{16\pi f^2} \left(C_s - \frac{C_t}{2} \right), \quad (\text{A.7})$$

$$M_1 = \frac{\hat{s}}{16\pi f^2} \frac{C_t}{6}. \quad (\text{A.8})$$

Eventually, the following conditions are obtained, respectively,

$$\frac{\hat{s}}{f^2} \lesssim \frac{16\pi}{|2C_s - C_t|}, \quad (\text{A.9})$$

$$\frac{\hat{s}}{f^2} \lesssim \frac{48\pi}{|C_t|}. \quad (\text{A.10})$$

If the condition, $|C_t| \leq 3|C_s| \leq 2|C_t|$, is satisfied, the unitarity bound given by the first modes of the partial wave amplitude is stronger than that given by the zeroth modes.

B Custodial symmetry of derivative interactions

Dimension six derivative interactions of Higgs doublets naively violate the custodial symmetry. We study conditions which ensure the symmetry on derivative interactions for 2HDMs. In the following discussion, we refer results and notations in Ref. [15].

Derivative interactions are classified into three kinds of operators with combinations of indices: operators including unique indices are called type I, e.g. $\partial(H_1^\dagger H_1)\partial(H_1^\dagger H_1)$; in type II, only one of four doublets has a different index, e.g. $\partial(H_1^\dagger H_1)\partial(H_1^\dagger H_2)$; the others belong to type III, e.g. $\partial(H_1^\dagger H_2)\partial(H_2^\dagger H_1)$. For type I and II, current-current interactions, namely O^T operators, violate the custodial symmetry because they produce additional contributions to the mass of the Z boson. This is interpreted from a different viewpoint with the nonlinear representation. For type I, as studied in Ref. [16], operators belonging to O^T consist of the generator of the hypercharge i.e. the third generator of $SU(2)_R$:

$$(hT^{R3}\partial h)(hT^{R3}\partial h) = \frac{1}{2}O_{1111}^T, \quad (\text{B.1})$$

where h is a real scalar multiplet which corresponds to the Higgs doublet. Since the generator violates $SO(4)$ symmetry, the custodial symmetry cannot be preserved after the EWSB. In other word, operators which consist of $SO(4)$ symmetric combinations of generators produce the custodial symmetric derivative interactions. It is also the case for operators of type II¹².

For type III derivative interactions, the situation is different, that is, certain combinations of $SU(2)_R$ violating operators recover $SU(2)_R$ symmetry. In this type, the following operators produce real DOF of derivative interactions:

$$T_{11}^{L\alpha}T_{22}^{L\alpha} = \frac{1}{4}(3O_{1221}^H - O_{1122}^T + O_{1221}^T), \quad (\text{B.2})$$

$$T_{12}^{L\alpha}T_{12}^{L\alpha} = \frac{1}{2}(3O_{1122}^H + 3O_{1212}^H + 3O_{2121}^H + O_{1122}^T - O_{1221}^T), \quad (\text{B.3})$$

$$T_{11}^{R\beta}T_{22}^{R\beta} = \frac{1}{4}(3O_{1212}^H + 3O_{2121}^H + O_{1122}^T - O_{1212}^T - O_{2121}^T), \quad (\text{B.4})$$

$$T_{12}^{R\beta}T_{12}^{R\beta} = \frac{1}{2}(3O_{1122}^H + 3O_{1221}^H - O_{1122}^T + O_{1212}^T + O_{2121}^T), \quad (\text{B.5})$$

$$S_{12}^{\alpha 3}S_{12}^{\alpha 3} = \frac{1}{2}(3O_{1122}^H - 3O_{1212}^H - 3O_{2121}^H + O_{1122}^T - O_{1221}^T), \quad (\text{B.6})$$

$$S_{12}^{\alpha\beta}S_{12}^{\alpha\beta} = \frac{3}{2}(2O_{1122}^H - O_{1221}^H - O_{1212}^H - O_{2121}^H), \quad (\text{B.7})$$

$$T_{11}^{R3}T_{22}^{R3} = \frac{1}{2}O_{1122}^T, \quad (\text{B.8})$$

$$T_{12}^{R3}T_{12}^{R3} = \frac{1}{2}(O_{1221}^T + O_{1212}^T + O_{2121}^T), \quad (\text{B.9})$$

$$U_{12}U_{12} = \frac{1}{2}(O_{1221}^T - O_{1212}^T - O_{2121}^T), \quad (\text{B.10})$$

¹²It is also true for imaginary DOF of derivative interactions. Any imaginary DOF of coefficients violate, in addition to the CP symmetry, the custodial symmetry. There are no relations so as to ensure the custodial symmetry for imaginary DOF, so that we discuss only real DOF here.

where $X_{ij}^A := (hX_{(i,j)}^A \partial h)$ and h includes eight real scalar fields interpreted as two Higgs doublets. Generators $T_{(i,j)}^{L\alpha}$, $T_{(i,j)}^{R\beta}$, $S_{(i,j)}^{\alpha\beta}$ and $U_{(i,j)}$ are respectively $(\mathbf{3}, \mathbf{1})$, $(\mathbf{1}, \mathbf{3})$, $(\mathbf{3}, \mathbf{3})$ and $(\mathbf{1}, \mathbf{1})$ representations of $SU(2)_L \times SU(2)_R$. Explicit forms of these matrices are given in Ref. [15]. If we naively follow the discussion for type I and II, coefficients of operators, (B.6), (B.8) and (B.9), apparently should vanish for $SO(4)$ symmetry. However, we have found that operators preserving $SU(2)_R$ can be given by several combinations of operators violating $SU(2)_R$. Since these operators are not linearly independent of each other, several relations are derived. In these relations, the above operators violating $SU(2)_R$ symmetry appear with a certain proportional relations:

$$a_{1212}^S : a_{1122}^Y : a_{1212}^Y = 1 : -2 : 1, \quad (\text{B.11})$$

where they are respectively coefficients of operators (B.6), (B.8) and (B.9). This condition is expressed as

$$c_{1122}^T + c_{1221}^T + c_{1212}^T = 0, \quad (\text{B.12})$$

$$3c_{1122}^T + c_{1212}^H - c_{1221}^H = 0, \quad (\text{B.13})$$

where $c_{ijkl}^{H,T}$ are defined as coefficients of $O_{ijkl}^{H,T}$. The result is consistent with the custodial symmetric conditions shown in Ref. [15].

The above analysis is easily extended to models including N Higgs doublets. In the case, two other classes of derivative interactions should be defined: operators including three deferent indices are called type IV, e.g. $\partial(H_i^\dagger H_j) \partial(H_i^\dagger H_k)$; the other operators whose indices are totally different each other are classified as type V, e.g. $\partial(H_i^\dagger H_j) \partial(H_k^\dagger H_l)$. With similar discussions given in the above, the following proportional relations are obtained:

$$a_{ijk}^S : a_{iij}^Y : a_{ijik}^Y = 1 : -2 : 1, \quad (\text{B.14})$$

$$a_{ijkl}^S : a_{ijkl}^S : a_{ijkl}^Y : a_{ikjl}^Y : a_{iljk}^Y = 1 : -1 : 1 : -2 : 1, \quad (\text{B.15})$$

$$a_{ikjl}^S : a_{iljk}^S : a_{ijkl}^Y : a_{ikjl}^Y : a_{iljk}^Y = 1 : 1 : -1 : 1 : 1, \quad (\text{B.16})$$

where a_{ijkl}^Y and a_{ijkl}^S are respectively coefficients $T_{ij}^{R3} T_{kl}^{R3}$ and $S_{ij}^{\alpha3} S_{kl}^{\alpha3}$, and the first relation is for type IV and the others for type V. The following relations are induced for coefficients of derivative interactions: for type IV,

$$c_{iijk}^T + c_{ijik}^T + c_{ijki}^T = 0, \quad (\text{B.17})$$

$$3c_{iijk}^T + c_{ijki}^H - c_{ijik}^H = 0; \quad (\text{B.18})$$

for type V,

$$c_{ijkl}^H - c_{ijlk}^H - c_{ikjl}^H + c_{iklj}^H + c_{iljk}^H - c_{ilkj}^H = 0, \quad (\text{B.19})$$

$$3(c_{ijkl}^T + c_{ijlk}^T) - c_{ikjl}^H + c_{iklj}^H - c_{iljk}^H + c_{ilkj}^H = 0, \quad (\text{B.20})$$

$$3(c_{ikjl}^T + c_{iklj}^T) - c_{ijkl}^H + c_{ijlk}^H + c_{iljk}^H - c_{ilkj}^H = 0, \quad (\text{B.21})$$

$$3(c_{iljk}^T + c_{ilkj}^T) + c_{ijkl}^H - c_{ijlk}^H + c_{ikjl}^H - c_{iklj}^H = 0. \quad (\text{B.22})$$

After imposing these conditions to ensure $SO(4)$ symmetry on derivative interactions, remained DOF of them are shown in Tab. 2.

C Unitarity matrices and other bounds

Following matrices are given by zeroth modes of partial wave amplitudes for various VBS processes in 2HDMs. Using the largest eigenvalue of them, the perturbative unitarity bound is obtained with Eq. (3.7).

	with	without
I	N	$2N$
II	$N(N-1)$	$2N(N-1)$
III	$N(N-1)$	$3N(N-1)$
IV	$N(N-1)(N-2)$	$3N(N-1)(N-2)$
V	$N(N-1)(N-2)(N-3)/6$	$N(N-1)(N-2)(N-3)/2$
Sum	$N^2(N^2+5)/6$	$N^2(N^2+3)/2$

Table 2: Real DOF of dimension six derivative interactions on models including N Higgs doublets with/without $SO(4)$ symmetry for each type.

C.1 Neutral two body states

The matrix for partial wave amplitudes of neutral two body states are shown here. Initial and final states are given by eight states, namely, $C_1^+ C_1^-$, $C_1^+ C_2^-$, $C_2^+ C_1^-$, $C_2^+ C_2^-$, $N_1 N_1^\dagger$, $N_1 N_2^\dagger$, $N_2 N_1^\dagger$ and $N_2 N_2^\dagger$. If all of coefficients are, except for c_{1111}^H , turned off, the matrix becomes the one in the case of 1HDM given in Eq. (2.5).

$$\left(\begin{array}{cccc}
 \frac{3c_{1111}^T + c_{1111}^H}{2} & \frac{3c_{1112}^T + c_{1112}^H}{2} & \frac{3c_{1112}^T + c_{1112}^H}{2} & \frac{3c_{1221}^T - c_{1221}^H + 2c_{1122}^H}{2} \\
 \frac{3c_{1112}^T + c_{1112}^H}{2} & \frac{3c_{1122}^T + 2c_{1221}^H - c_{1122}^H}{2} & \frac{3c_{1212}^T + c_{1212}^H}{2} & \frac{3c_{2221}^T + c_{2221}^H}{2} \\
 \frac{3c_{1112}^T + c_{1112}^H}{2} & \frac{3c_{1212}^T + c_{1212}^H}{2} & \frac{3c_{1122}^T + 2c_{1221}^H - c_{1122}^H}{2} & \frac{3c_{2221}^T + c_{2221}^H}{2} \\
 \frac{3c_{1221}^T - c_{1221}^H + 2c_{1122}^H}{2} & \frac{3c_{2221}^T + c_{2221}^H}{2} & \frac{3c_{2221}^T + c_{2221}^H}{2} & \frac{3c_{2222}^T + c_{2222}^H}{2} \\
 c_{1111}^H & c_{1112}^H & c_{1112}^H & c_{1122}^H \\
 c_{1112}^H & c_{1221}^H & c_{1212}^H & c_{2221}^H \\
 c_{1112}^H & c_{1212}^H & c_{1221}^H & c_{2221}^H \\
 c_{1122}^H & c_{2221}^H & c_{2221}^H & c_{2222}^H \\
 c_{1111}^H & c_{1112}^H & c_{1112}^H & c_{1122}^H \\
 c_{1112}^H & c_{1221}^H & c_{1212}^H & c_{2221}^H \\
 c_{1112}^H & c_{1212}^H & c_{1221}^H & c_{2221}^H \\
 c_{1122}^H & c_{2221}^H & c_{2221}^H & c_{2222}^H \\
 \frac{3c_{1111}^T + c_{1111}^H}{2} & \frac{3c_{1112}^T + c_{1112}^H}{2} & \frac{3c_{1112}^T + c_{1112}^H}{2} & \frac{3c_{1221}^T - c_{1221}^H + 2c_{1122}^H}{2} \\
 \frac{3c_{1112}^T + c_{1112}^H}{2} & \frac{3c_{1122}^T + 2c_{1221}^H - c_{1122}^H}{2} & \frac{3c_{1212}^T + c_{1212}^H}{2} & c_{2221}^T \\
 \frac{3c_{1112}^T + c_{1112}^H}{2} & \frac{3c_{1212}^T + c_{1212}^H}{2} & \frac{3c_{1122}^T + 2c_{1221}^H - c_{1122}^H}{2} & c_{2221}^T \\
 \frac{3c_{1221}^T - c_{1221}^H + 2c_{1122}^H}{2} & c_{2221}^T & c_{2221}^T & \frac{3c_{2222}^T + c_{2222}^H}{2}
 \end{array} \right) \cdot \quad (C.1)$$

If we impose $SO(4)$ symmetry on the above matrix and eliminate c_{1122}^T and c_{1212}^T with Eqs. (B.12) and (B.13),

the matrix is simplified as follows:

$$\left(\begin{array}{cccc}
 \frac{c_{1111}^H}{2} & \frac{c_{1112}^H}{2} & \frac{c_{1112}^H}{2} & \frac{3c_{1221}^T - c_{1221}^H + 2c_{1122}^H}{2} \\
 \frac{c_{1112}^H}{2} & \frac{c_{1221}^H + c_{1212}^H - c_{1122}^H}{2} & -\frac{3c_{1221}^T - c_{1221}^H}{2} & \frac{c_{2221}^H}{2} \\
 \frac{c_{1112}^H}{2} & -\frac{3c_{1221}^T - c_{1221}^H}{2} & \frac{c_{1221}^H + c_{1212}^H - c_{1122}^H}{2} & \frac{c_{2221}^H}{2} \\
 \frac{3c_{1221}^T - c_{1221}^H + 2c_{1122}^H}{2} & \frac{c_{2221}^H}{2} & \frac{c_{2221}^H}{2} & \frac{c_{2222}^H}{2} \\
 c_{1111}^H & c_{1112}^H & c_{1112}^H & c_{1122}^H \\
 c_{1112}^H & c_{1221}^H & c_{1212}^H & c_{2221}^H \\
 c_{1112}^H & c_{1212}^H & c_{1221}^H & c_{2221}^H \\
 c_{1122}^H & c_{2221}^H & c_{2221}^H & c_{2222}^H \\
 c_{1111}^H & c_{1112}^H & c_{1112}^H & c_{1122}^H \\
 c_{1112}^H & c_{1221}^H & c_{1212}^H & c_{2221}^H \\
 c_{1112}^H & c_{1212}^H & c_{1221}^H & c_{2221}^H \\
 c_{1122}^H & c_{2221}^H & c_{2221}^H & c_{2222}^H \\
 \frac{c_{1111}^H}{2} & \frac{c_{1112}^H}{2} & \frac{c_{1112}^H}{2} & \frac{3c_{1221}^T - c_{1221}^H + 2c_{1122}^H}{2} \\
 \frac{c_{1112}^H}{2} & \frac{c_{1221}^H + c_{1212}^H - c_{1122}^H}{2} & -\frac{3c_{1221}^T - c_{1221}^H}{2} & 0 \\
 \frac{c_{1112}^H}{2} & -\frac{3c_{1221}^T - c_{1221}^H}{2} & \frac{c_{1221}^H + c_{1212}^H - c_{1122}^H}{2} & 0 \\
 \frac{3c_{1221}^T - c_{1221}^H + 2c_{1122}^H}{2} & 0 & 0 & \frac{c_{2222}^H}{2}
 \end{array} \right). \quad (C.2)$$

C.2 Singly charged two body states

The matrix for zeroth mode partial wave amplitudes of singly charged two body states is shown below. Initial and final states consist of the following four states: $C_1^+ N_1^\dagger$; $C_1^+ N_2^\dagger$; $C_2^+ N_1^\dagger$ and $C_2^+ N_2^\dagger$:

$$\left(\begin{array}{cccc}
 -\frac{3c_{1111}^T + c_{1111}^H}{2} & -\frac{3c_{1112}^T + c_{1112}^H}{2} & -\frac{3c_{1112}^T + c_{1112}^H}{2} & -\frac{3c_{1221}^T + c_{1221}^H}{2} \\
 -\frac{3c_{1112}^T + c_{1112}^H}{2} & -\frac{3c_{1122}^T + c_{1122}^H}{2} & -\frac{3c_{1212}^T + c_{1212}^H}{2} & -\frac{3c_{2221}^T + c_{2221}^H}{2} \\
 -\frac{3c_{1112}^T + c_{1112}^H}{2} & -\frac{3c_{1212}^T + c_{1212}^H}{2} & -\frac{3c_{1122}^T + c_{1122}^H}{2} & -\frac{3c_{2221}^T + c_{2221}^H}{2} \\
 -\frac{3c_{1221}^T + c_{1221}^H}{2} & -\frac{3c_{2221}^T + c_{2221}^H}{2} & -\frac{3c_{2221}^T + c_{2221}^H}{2} & -\frac{3c_{2222}^T + c_{2222}^H}{2}
 \end{array} \right). \quad (C.3)$$

We explicitly impose $SO(4)$ symmetry like the case of the neutral states:

$$\begin{pmatrix} -\frac{c_{1111}^H}{2} & -\frac{c_{1112}^H}{2} & -\frac{c_{1112}^H}{2} & -\frac{3c_{1221}^T+c_{1221}^H}{2} \\ -\frac{c_{1112}^H}{2} & \frac{c_{1221}^H-c_{1212}^H-c_{1122}^H}{2} & \frac{3c_{1221}^T-c_{1221}^H}{2} & -\frac{c_{2221}^H}{2} \\ -\frac{c_{1112}^H}{2} & \frac{3c_{1221}^T-c_{1221}^H}{2} & \frac{c_{1221}^H-c_{1212}^H-c_{1122}^H}{2} & -\frac{c_{2221}^H}{2} \\ -\frac{3c_{1221}^T+c_{1221}^H}{2} & -\frac{c_{2221}^H}{2} & -\frac{c_{2221}^H}{2} & -\frac{c_{2222}^H}{2} \end{pmatrix}. \quad (C.4)$$

C.3 Doubly charged two body states

Entries of the following matrices are coefficients of VBS processes for doubly charged states: $C_1^+ C_1^+$; $C_1^+ C_2^+$ and $C_2^+ C_2^+$. If processes include the same particle states in their final states, i.e. the first and the third ones, the unitarity bound becomes weak as mentioned in App. A. The effect has been included in the following matrices:

$$\begin{pmatrix} -\frac{c_{1111}^T-c_{1111}^H}{2} & c_{1112}^H - c_{1112}^T & -\frac{c_{1212}^T-c_{1212}^H}{2} \\ -\frac{c_{1112}^T-c_{1112}^H}{2} & -\frac{3c_{1221}^T+3c_{1122}^T+c_{1221}^H+c_{1122}^H}{2} & -\frac{c_{2221}^T-c_{2221}^H}{2} \\ -\frac{c_{1212}^T-c_{1212}^H}{2} & c_{2221}^H - c_{2221}^T & -\frac{c_{2222}^T-c_{2222}^H}{2} \end{pmatrix}. \quad (C.5)$$

We also impose $SO(4)$ symmetry on the above one:

$$\begin{pmatrix} \frac{c_{1111}^H}{2} & c_{1112}^H & \frac{3c_{1221}^T-c_{1221}^H+4c_{1212}^H}{6} \\ \frac{c_{1112}^H}{2} & -\frac{3c_{1221}^T+c_{1212}^H+c_{1122}^H}{2} & \frac{c_{2221}^H}{2} \\ \frac{3c_{1221}^T-c_{1221}^H+4c_{1212}^H}{6} & c_{2221}^H & \frac{c_{2222}^H}{2} \end{pmatrix}. \quad (C.6)$$

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